Covariance matrix for the measurement of protons from the $pp \to pp\eta$ reaction at Q = 15.5 MeV using COSY-11

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At the COSY-11 facility the identification of the $pp \to pp\eta$ reaction is based on the measurement of the momentum vectors of the outgoing protons and the utilisation of the missing mass technique [1]. Inaccuracy of the momentum determination manifests itself in the population of kinematically forbidden regions of the phase space, preventing the precise comparison of the theoretically derived and experimentally determined differential cross sections. Figure 1(left) visualizes this effect and clearly demonstrates that the data scatter significantly outside the kinematically allowed region, in spite of the fact that the precision of the fractional momentum determination in the laboratory system ($\sigma(p_{lab})/p_{lab} \approx 7 \cdot 10^{-3}$) is quite high. Therefore, when seeking for small effects like for example the influence of the proton-η interaction on the population density of the phase-space, one needs either to fold theoretical calculations with the experimental resolution, or to perform the kinematical fitting of the data. Both procedures require the knowledge of the covariance matrix, and thus its determination constitutes a necessary step in the differential analysis and interpretation of the $pp \to pp\eta$ reaction measured with high statistics ($\approx$ 24000 events) at Q = 15.5 MeV.

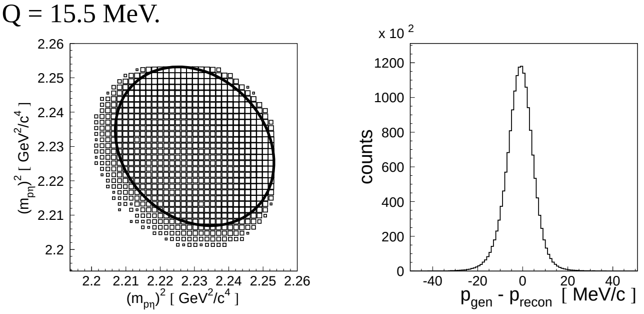

Fig. 1: **(left)** Dalitz plot distribution of the $pp \to pp\eta$ reaction simulated at Q = 15.5 MeV. The number of entries is shown in a logarithmic scale. The solid line depicts the kinematically available area. **(right)** Spectrum of differences between reconstructed and generated absolut momentum of protons. Both pictures show results obtained taking into account the experimental conditions as described in the text.

In order to derive the covariance matrix we need to recognize and quantify all possible sources of errors in the reconstruction of the protons momenta. The four dominant effects are: i) multiple scattering in the dipole exit foil, air and detectors, ii) finite resolution of the position determination of the drift chambers, iii) finite distributions of the beam momentum and of the reaction points, and iv) the possible wrong assignment of hits to the particle tracks in drift chambers in the case of the close tracks. Some of these, like eg. the multiple scattering, depend on the outgoing proton momenta, others, like the beam momentum distribution, depend on the specific run conditions and therefore must be determined for each run separately. Here we report on the estimations made for the measurement of the $pp \to pp\eta$ reaction at Q = 15.5 MeV.

From the distributions of the elastically scattered protons, the Schottky frequency spectrum, and the missing mass distribution of the $pp \to ppX$ reaction, using a method described elsewhere [2], we have estimated that for the measurement under discussion the spread of the beam momentum (at $p_{beam} = 2.0259\ GeV/c$) and the reaction points in horizontal and vertical direction amount to $\sigma(p_{beam}) = 0.63 \pm 0.03$ MeV/c, $\sigma(x) = 0.22 \pm 0.02$ cm, and $\sigma(y) = 0.38 \pm 0.04$ cm, respectively.

In order to estimate the variances and covariances for all possible combinations of the momentum components for two registered protons we have generated 10^8 $pp \to pp\eta$ events and simulated the response of the COSY-11 detection setup taking into account the above mentioned factors and the known resolutions of the detector components. Next we analysed the signals by means of the same reconstruction procedure as used in case of the experimental data. Covariances between the i^{th} and the j^{th} components of the event vector ($P = [p_{1x}, p_{1y}, p_{1z}, p_{2x}, p_{2y}, p_{2z}]$) were established as the average of the product of the deviations between the reconstruced and generated values. The explicit formula for the sample of N reconstructed events reads:

$$cov(i,j) = \frac{1}{N}\sum_{k=1}^{N}(P^k_{i,gen} - P^k_{i,recon})(P^k_{j,gen} - P^k_{j,recon}), \quad (1)$$

where $P^k_{i,gen}$ and $P^k_{i,recon}$ denote the generated and reconstructed values for the i^{th} component of the vector P describing the k^{th} event.

Because of the inherent symmetries of the covariance ($cov(i,j) = cov(j,i)$) and the indistinguishability of the registered protons, there are only 12 independent values which determine the 6 x 6 error matrix V unambiguously. They are listed below in units of MeV^2/c^2, as established in the laboratory system with the z-coordinate parallel to the beam axis and y-coordinate corresponding to the vertical direction.

$$V = \begin{array}{c} \begin{array}{cccccc} p_{1x} & p_{1y} & p_{1z} & p_{2x} & p_{2y} & p_{2z} \end{array} \\ \left[\begin{array}{cccccc} 6.1 & -0.1 & -15.4 & 2.0 & 0.0 & -3.9 \\ - & 6.8 & 0.1 & - & 0.0 & 0.0 \\ - & - & 43.0 & - & - & 7.4 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \end{array}\right] \end{array} \begin{array}{c} \\ p_{1x} \\ p_{1y} \\ p_{1z} \\ p_{2x} \\ p_{2y} \\ p_{2z} \end{array} \quad (2)$$

Since the measurements have been performed close to the kinematical threshold the ejectile momentum component parallel to the beam is by far the largest one and its variance ($var(p_z) = 43\ MeV^2/c^2$) determines in the first order the error of the momentum measurement. The second largest contribution stems from an anti-correlation between the $x-$ and $z-$ momentum components ($cov(p_x, p_z) = -15.4\ MeV^2/c^2$), which is due to the bending of the proton trajectory – mainly in the horizontal direction – inside a COSY-11 dipole magnet. There is also a significant correlation between the z components of different protons which is due to the smearing of the reaction points, namely, if in the analysis the assumed reaction point deviates from the actual one, a mistake made in the reconstruction affects both protons similarly.

Taking into account components of the covariance matrix V and the distribution of the proton momenta for the $pp \to pp\eta$ reaction at Q = 15.5 MeV results in an average error for the measurement of the proton momentum of about 6 MeV/c. The corresponding distribution is shown in figure 1(right).

References:

[1] S. Brauksiepe et al., NIM **A 376** (1996) 397.
[2] P. Moskal et al., NIM **A 466** (2001) 448.