A method for the luminosity determination in the quasi-free meson production reactions

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A method for luminosity determination in the quasi-free meson production was developed for the COSY-11 experiment on the production of the η meson in the quasi-free proton-neutron collisions, close to the kinematical threshold [1]. However, this method is of universal nature and may be applied to determine the luminosity in any quasi-free production reaction.

The reference reaction chosen for the purpose of calculation of the luminosity was the quasi-free $pp \to pp$ scattering. For the free proton-proton scattering the luminosity could be determined as a normalization constant between the measured angular distribution of the cross section and the corresponding spectrum extracted from the previous experiments. For the quasi-free reaction the evaluation becomes more complicated due to the Fermi motion of nucleons inside the nucleus. Since the direction and momentum of the bound nucleon varies from event to event this implies that the direction of the center-of-mass velocity of the colliding nucleons as well as the total available energy for the reaction also varies from event to event. Therefore, protons registered in the laboratory under a given scattering angle or at a given part of the detection system correspond to the finite range of scattering angles in the proton-proton center-of-mass frame. This implies that the experimental angular distributions cannot be directly compared to literature values. Instead an evaluation of the luminosity requires simulations taking into account the Fermi motion of the nucleons, and the variations of differential cross sections for the elastic scattering as a function of the scattering angle and energy.

For each simulated event we know the generated Fermi momentum of the nucleon, as well as the scattering angle of protons in their center-of-mass system. This permits us to assign to each event a weight corresponding to the differential cross section, which is uniquely determined by the scattering angle and the total collision energy $\sqrt{s}$.

For free proton-proton scattering we could measure the number of events - $\Delta N(\theta,\phi)$ scattered into the solid angle $\Delta\Omega(\theta,\phi)$ around the polar and azimuthal angles θ and φ, respectively. In this case the angles in laboratory and in the center-of-mass systems are univocally related to each other. With the known differential cross section for proton-proton scattering into that particular solid angle, and having known the value of the solid angle $\Delta\Omega(\theta,\phi)$ from the Monte-Carlo simulations the luminosity can be calculated according to the formula:

$$L = \frac{\Delta N(\theta,\phi)}{\Delta\Omega(\theta,\phi)\frac{d\sigma}{d\Omega}(\theta,\phi)}. \quad (1)$$

In the case of quasi-free proton-proton scattering the number of elastically scattered protons ΔN into a solid angle $\Delta\Omega(\theta_{lab},\phi_{lab})$ is proportional to L – the integrated luminosity and also to the inner product of the differential cross section for scattering into the solid angle around θ^* and ϕ^* angles – $\frac{d\sigma}{d\Omega}(\theta^*,\phi^*,p_F,\theta_F,\phi_F)$ – and the probability density of the distribution of the Fermi momentum $f(p_F,\theta_F,\phi_F)$:

$$\Delta N_{exp}(\Delta\Omega(\theta_{lab},\phi_{lab})) = L\int_{\Delta\Omega(\theta_{lab},\phi_{lab})}\frac{d\sigma}{d\Omega}(\theta^*,\phi^*,p_F,\theta_F,\phi_F)\,f(p_F,\theta_F,\phi_F)\,dp_F\,d\cos\theta_F\,d\phi_F\,d\phi^*\,d\cos\theta^*. \quad (2)$$

The angles θ^* and ϕ^* are expressed in the proton-proton center-of-mass system, while the angles θ_{lab} and ϕ_{lab} describe the directions in the laboratory. In the case of the complex detection geometry, with a magnetic field a solid angle corresponding to a given part of the detector cannot, in general, be expressed in a closed analytical form. Therefore, the integral in Equation 2 must be computed using the Monte-Carlo method. For the evaluation of a given event by the Monte Carlo program, first we choose randomly the momentum of a nucleon inside a deuteron according to the Fermi distribution. Next, the total energy $\sqrt{s}$ for the proton-proton scattering and the vector of the center-of-mass velocity are determined. Then, assuming an isotropic distribution we generate the momentum of protons in the proton-proton center-of-mass frame. Further on, according to the generated angle and the total energy $\sqrt{s}$ we assign to the event a probability equal to the differential cross section, utilizing the bilinear interpolation in the $\sqrt{s}$–θ^* space. Next, the momenta of protons are transformed to the laboratory frame and are used as an input in the simulation of the detector signals with the use of the GEANT package.

In order to calculate the integral on the right hand side of this equation we simulated N_0 events according to the procedure described above. Due to the weights assigned to the events the integral is not dimensionless and its units correspond to the units of the cross section used for the calculations. The number obtained from the Monte-Carlo simulations must be then normalized such that the integral over the full solid angle equals the cross section for the elastic scattering averaged over the distribution of the total reaction energy $\sqrt{s}$ resulting from the Fermi distribution of the target nucleon. This means that we need to divide the resultant Monte-Carlo integral by the number of generated events N_0 and multiply it by the factor of 2π. A factor of 2π comes from the normalization of the differential cross section, regarding the fact that protons taking part in the scattering are indistinguishable. Hence, the formula for the calculation of the integrated luminosity for the quasi-free reaction reads:

$$L = \frac{N_0 \Delta N_{exp}}{2\pi I}, \quad (3)$$

where the normalization constant $N_0/2\pi$ is subject to the Monte-Carlo method used for the integral computation, and I in the denominator denotes the integral obtained from simulation of N_0 events.

ΔN_{exp} from Equation 2 can be determined as a number of elastically scattered protons registered in a given part of the detection system.

A more detailed description of the method described above and its application to the calculation of the luminosity for the measurement of the quasi-free $pn \to pn\eta$ reaction can be found in [2].

References:

[1] P. Moskal et al., J. Phys. **G 32** (2006) 629.
[2] P. Moskal, R. Czyżykiewicz, AIP Conf. Proc. **950** (2007) 118.