Missing Mass Resolution in Near–Threshold Measurements

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In the COSY–11 experiment, the identification of mesons produced in proton–proton collisions is performed using the missing mass method. It was observed, that the missing mass distribution always broadens with increasing excess energy (see example of missing mass spectra shown in Fig. 1). This behaviour is due to a kinematical effect: In the CM–system, the missing mass squared can be written as:

$$MM^2 = (E_{TOT} - E_1 - E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2, \tag{1}$$

where E_{TOT} is the total energy, E_1, E_2 are the energies of the two outgoing protons, and $\vec{p}_1, \vec{p}_2$ are their momentum vectors. In the non–relativistic limit this equation changes to:

$$MM^2 = m_X^2 + 2m_X \cdot \varepsilon - 2m_X \cdot \frac{p_1^2 + p_2^2}{2m_p} - (\vec{p}_1 + \vec{p}_2)^2, \tag{2}$$

where m_X is the mass of produced meson, m_p is the proton mass and ε is the excess energy. The finite width of the missing mass peak is caused by limited experimental precision of the measured momenta $\vec{p}_1$, $\vec{p}_2$, and, therefore, can be expressed as:

$$(\Delta(MM^2))^2 = \sqrt{\sum_{i=1}^{3} \left((\frac{\delta MM^2}{\delta p_{1i}} \Delta p_{1i})^2 + (\frac{\delta MM^2}{\delta p_{2i}} \Delta p_{2i})^2 \right)}, \tag{3}$$

where i numerates the three components of the momentum vectors. We assume, that the experimental uncertainties of the momentum vectors (Δp_{1i} and Δp_{2i}) are all equal to some constant value Δp. The partial derivatives in eq. 3 are calculated by differentiation of eq. 2. Next, averaging $(\Delta(MM^2))^2$ over the phase space for the outgoing protons causes e.g. that products $\vec{p}_1 \cdot \vec{p}_2$ disappear, and after some algebra one obtains:

$$(\Delta(MM^2))^2 = \left[4(\frac{m_X}{m_p} + 1)^2 + 4 \right] \cdot \frac{2m_p m_X}{m_p m_X} \cdot \varepsilon \cdot (\Delta p)^2 \tag{4}$$

The width of the peak in the missing mass squared spectrum is thus proportional to $\sqrt{\epsilon}$:

$$\Delta(MM^2) = a \cdot \sqrt{\varepsilon} \tag{5}$$

As shown in Fig. 2, this formula reproduces the energy dependence of the width of the η missing mass peak observed in the $pp \to pp\eta$ measurement well.

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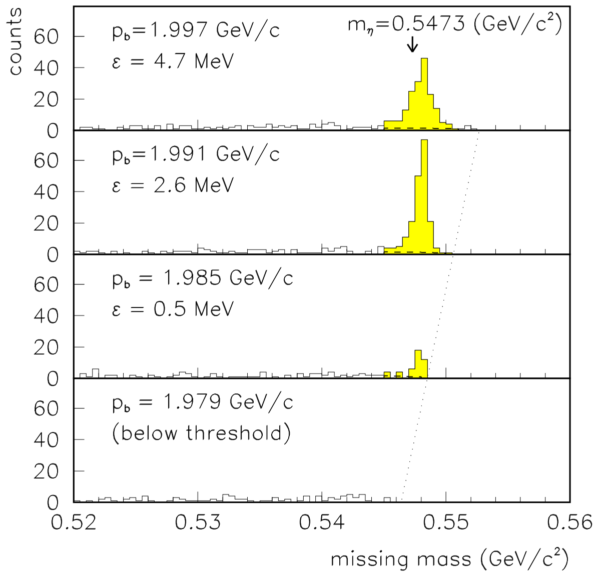

Figure 1: Missing mass of the proton–proton system measured for three different momenta above and one momentum below threshold for η meson production. The dotted line indicates the kinematical missing mass limit.

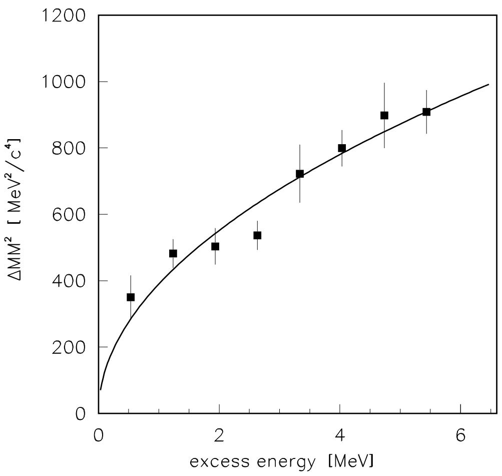

Figure 2: Width of the η missing mass peak as a function of the excess energy. The solid line corresponds to a fit using eq. 5 with the parameter $a = 390\sqrt{MeV^3}/c^4$.