S-wave η'-proton FSI; phenomenological analysis of near-threshold production of π^0, η, and η' mesons in proton-proton collisions.

P. Moskal[1,2], H.-H. Adam[3], J.T. Balewski[4,5], A. Budzanowski[4], J. Budziński[1], D. Grzonka[2], L. Jarczyk[1], A. Khoukaz[3], K. Kilian[2], P. Kowina[2,6], N. Lang[3], T. Lister[3], W. Oelert[2], C. Quentmeier[3], R. Santo[3], G. Schepers[2,3], T. Sefzick[2], S. Sewerin[2], M. Siemaszko[6], J. Smyrski[1], A. Strzałkowski[1], M. Wolke[2], P. Wüstner[7], W. Zipper[6]

[1] *Institute of Physics, Jagellonian University, PL-30-059 Cracow, Poland*

[2] *IKP, Forschungszentrum Jülich, D-52425 Jülich, Germany*

[3] *IKP, Westfälische Wilhelms–Universität, D-48149 Münster, Germany*

[4] *Institute of Nuclear Physics, PL-31-342 Cracow, Poland*

[5] *IUCF, Bloomington, Indiana, IN-47405, USA*

[6] *Institute of Physics, University of Silesia, PL-40-007 Katowice, Poland*

[7] *ZEL, Forschungszentrum Jülich, D-52425 Jülich, Germany*

(April 17, 2000)

Abstract

We describe a novel technique for comparing total cross sections for the reactions $pp \to pp\pi^0$, $pp \to pp\eta$, and $pp \to pp\eta'$ close to threshold. The initial and final state proton-proton interactions are factored out of the total cross section, and the dependence of this reduced cross section on the volume of phase space is discussed. Different models of the proton-proton interaction are compared. We argue that the scattering length of the S-wave η'-proton interaction is of the order of 0.1 fm.

PACS: 13.60.Le, 13.75.-n, 13.85.Lg, 25.40.-h, 29.20.Dh

New results on η and η' meson production in the reaction $pp \to ppX$, measured very recently at the COSY-11 facility [1,2], together with previous data [3–8], determine the energy dependence of the near-threshold total cross section with a precision comparable to the measurements of the reaction $pp \to pp\pi^0$ [9,10]. These new data encouraged us to perform a phenomenological analysis similar to those of references [11–13]. Here we concentrate on π^0, η, and η' meson production, and complete the analysis of these references by taking into account the interaction between the incident protons, and by introducing a new representation of the data. The production rates of π^0, η, and η' mesons will be compared as a function of the available phase space. We will study the phase-space dependence of the quantity $|M_0|$ which is derived from the total cross section with the ISI and pp-FSI factored out. Consideration of the dependence of $|M_0|$ on phase space allows us to infer the η-proton and η'-proton interactions. To avoid large ambiguities due to differences in pp-FSI models, we normalize the $|M_0|$ for η and η' mesons to the one for the π^0 meson and show that the resulting η'-proton interaction is comparable to the π^0-proton one.

In general, the total cross section is presented as a function of the dimensionless parameter η_M [9,10,12] [1], which is defined as the maximum meson momentum in units of meson mass ($\eta_M = \frac{q_{max}}{M}$), or as a function of the center-of-mass excess energy Q [3,6,8], where nonrelativistically these variables are related by:

$$\eta_M^2 = \frac{4\ m_p}{2\ m_p\ M + M^2}\ Q, \tag{1}$$

with m_p and M denoting the proton and meson mass, respectively. The above equation shows that the proportionality factor between the variables Q and η_M^2 changes for different mesons, since the masses of the π^0, η, and η' are distinct. Hence, depending which variable is selected, the relation between the cross section values changes for different mesons. For example, as shown in reference [4] the η meson production-cross-section exceeds the π^0 cross

[1]In order to avoid ambiguities with the abbreviation for the eta-meson, we introduce an additional suffix M for this parameter, which usually is called η.

section by about a factor of five using η_M, whereas the π^0 meson cross section is always larger when employing the Q scale.

The total cross section for $pp \to ppX$ is in general an integral over phase space, weighted by the square of the transition matrix element and normalized to the incoming flux factor F:

$$\sigma_{pp\to ppX} = \frac{1}{F}\int dV_{ps}\ |M_{pp\to ppX}|^2, \tag{2}$$

where X stands for the π^0, η or η' meson, V_{ps} denotes the phase space volume, and $\mathrm{F} = 2\ (2\pi)^5\sqrt{s\ (s\ -\ 4m_p^2)}$ [14], with s being the square of the total energy in the center-of-mass frame. The transition matrix element for the reaction $pp \to ppX$, $M_{pp\to ppX}$, incorporates the production mechanism and both the initial (ISI) and final (FSI) state interactions. In analogy with the *Watson-Migdal* approximation [18] for two body processes, we may assume that the complete transition amplitude for a production process $M_{pp\to ppX}$ factorizes approximately as [19]:

$$|M_{pp\to ppX}|^2 \approx |M_0|^2 \cdot |M_{FSI}|^2 \cdot ISI, \tag{3}$$

where M_0 represents the total production amplitude, M_{FSI} describes the elastic interaction among particles in the exit channel, and ISI denotes the reduction factor due to the interaction of the colliding protons.

Equation (2) suggests that a natural variable for comparing the total cross sections for different mesons may be the volume of available phase space,[2] $V_{ps} = \int dV_{ps}$. Figure 1 shows the yield of π^0, η, and η' mesons in the proton-proton interaction as a function of available phase space volume. The yield is defined as the cross section multiplied by the corresponding flux factor, and divided by the ISI factor. Close to threshold the initial state interaction, which reduces the total cross section, is dominated by proton-proton scattering

---

[2] In the nonrelativistic approximation, Q << M (which is justified at threshold), V_{ps} is proportional to the fourth power of the variable η_M or the square of the excess energy Q; $V_{ps} = \int dV_{ps} = \eta_M^4\ \sqrt{M + 2\ m_p} \cdot m_p^{-1}\ M^{\frac{5}{2}}\ \pi^3\ 2^{-5}$

in the $^3P_0$ state. As shown by Hanhart and Nakayama [15], this ISI may be estimated in terms of phase shifts and inelasticities. The ISI factor is close to unity for pion production, and amounts to ~ 0.2 [15] and ~ 0.33 [16] respectively for η and η' mesons at threshold. As shown in reference [15] the initial state interaction may be taken into account by multiplying the theoretical cross section by these factors. This justifies our choice of the dimensionless quantity $\frac{\sigma \cdot F}{ISI}$, which depends only on the primary production amplitude M_0 and on the final state interaction among the produced particles. Numerically we have confirmed that within the present model the effect of the pp-FSI is approximately independent of the produced meson mass throughout the phase space volume under investigation. In fact, the difference of the effect on the pp-FSI is about 1% for η and η' production and it is only about 10% larger than for the case of the π^0 production. This is in line with the G. Fäldt and C. Wilkin model which predict that the pp-FSI depends on the excess energy only [17].

Thus, the observed differences in the production yield for the different mesons, as presented in Figure 1, may be attributed directly to the square of the primary production amplitude $|M_0|^2$ convoluted with the FSI of the particles in the exit channel. Comparing the cross sections with flux-factor and ISI corrections one can see that over the relevant range of V_{ps} the dynamics for η' meson production is about six times weaker than for the π^0 meson, which again is a further factor of six weaker than that of the η meson.

Employing equations (2) and (3) and given the two additional assumptions that in the exit channel only the proton-proton interaction is significant ($M_{FSI} = M_{pp\to pp}$), and that the primary production amplitude is constant over the studied range of phase space, it is possible to calculate the quantity $|M_0|$. From this point we no longer refer to M_0 as the primary production amplitude, because the assumption ($M_{FSI} = M_{pp\to pp}$) implicitly shifts the proton-meson FSI from $|M_{FSI}|$ to $|M_0|$.

To evaluate $|M_0|$ we considered three possible parametrizations of the pp-FSI enhancement factor $|M_{pp\to pp}|^2$, which are presented in Figure 2. The solid line shows the squared proton-proton amplitude [20]:

$$M_{pp\to pp} = \frac{e^{\delta_{pp}(^1S_0)} \cdot \sin\delta_{pp}(^1S_0)}{C \cdot k}, \tag{4}$$

where $C^2 = \frac{2\pi\eta_c}{e^{2\pi\eta_c}-1}$ is the Coulomb penetration factor [21], η_c is the relativistic Coulomb parameter $\eta_c = \alpha/v$, with α - the fine structure constant and v - the proton velocity in the rest frame of the other proton. The phase-shifts $\delta_{pp}(^1S_0)$ are calculated according to the modified Cini-Fubini-Stanghellini formula including the Wong-Noyes Coulomb correction [22–24],

$$C^2\ p\ ctg\delta_{pp} + 2\ p\ \eta_c\ h(\eta_c) = -\frac{1}{a_{pp}} + \frac{b_{pp}\ p^2}{2} - \frac{P_{pp}\ p^4}{1+Q_{pp}\ p^2}, \tag{5}$$

where $h(\eta_c) = -ln(\eta_c) - 0.57721 + \eta_c^2\ \sum_{n=1}^{\infty} \frac{1}{n\cdot(n^2+\eta_c^2)}$ [25]. The phenomenological quantities $a_{pp} = -7.83$ fm and $b_{pp} = 2.8$ fm denote the scattering length and effective range [22], respectively. The parameters $P_{pp} = 0.73$ fm^3 and $Q_{pp} = 3.35$ fm^2 are related to the detailed shape of the nuclear potential and derived from a one-pion-exchange model [22]. These calculations give values which are in a good agreement with the phase shifts of the VPI partial wave analysis [26], shown as solid circles, and with the phase shifts of the Nijmegen analysis [27], shown as open squares.

The dashed line presents the enhancement from the proton-proton interaction, $|M_{pp\to pp}|^2$, estimated as an inverse of the squared Jost function, with the Coulomb interaction included [28]. In this case $|M_{pp\to pp}|^2$ is a dimensionless factor which approaches zero as the relative proton momentum k$\to$0, peaks sharply at k $\approx$ 25 MeV/c, and asymptotically approaches unity for large relative proton-proton momentum. The solid and dashed lines agree quite well for small relative protons momentum.

The dotted line corresponds to the inverse of the squared Jost function, calculated using the formulas of references [29,30], also corrected for the Coulomb force. The presented prescriptions evidently differ significantly, especially for k larger than $\approx$ 50 MeV/c.

The curves in Figure 2 are arbitrarily normalized to the same maximum value as found in reference [28]. Note that the values of $|M_0|$ extracted will depend on the absolute values of the enhancement factor $|M_{pp\to pp}|^2$, which is not well established. For example, the formula given by Druzhinin et al. [28] leads to a maximum value of $|M_{pp\to pp}|^2$ of about 50, whereas the authors of reference [11] find a value of about 20. Due to this wide variation, in the following we will discuss only the relative phase space dependence of $|M_0|$, rather than the absolute magnitude.

Figure 3 compares values extracted for $|M_0|$ near threshold production of π^0, η, and η' mesons, using $M_{pp\to pp}$ as shown in Figure 2 by the solid line. $|M_0|$ is arbitrarily normalized to unity for large V_{ps}, separately for each meson. If the assumptions used in the derivation of $|M_0|$ were strictly satisfied the values obtained would be equal to one as depicted by the solid line. It can be seen, however, that in the case of the η meson, $|M_0|$ grows with decreasing phase space volume. The observed deviation from unity is too large to be plausibly assigned to a variation in the primary production amplitude; the calculations of Moalem et al. [31] show that the primary production amplitude should change only by a few per cent in this energy range. Therefore, the observed behaviour of $|M_0|$ may be attributed to an attractive η-proton interaction, which was neglected in its derivation. The results for the π^0 and η' production, apart from the points closest to threshold[3], show that $|M_0|$ is indeed effectively constant over the region of the phase space studied, indicating that both the π^0-proton and η'-proton FSIs are too weak to be observable at current experimental accuracy. In the case of the π^0 this result was expected, since the S-wave π-proton interaction is much weaker than the η-proton one. The real part of the scattering length, $a_{p\pi} = 0.13$ fm [32], is about six times smaller than $a_{p\eta} = 0.75$ fm [33]. Regarding the η' meson, there is no experimental evidence for the η'-proton interaction except for a very conservative upper limit on the real value of

[3]Due to the steep decrease of the total cross section near threshold, a small change in the energy (0.2 MeV) lifts the points to significantly higher values. Moreover, at very low energies, nuclear and Coulomb scattering are expected to compete. The limit is at approximately 0.8 MeV proton energy in the rest frame of the other proton, where the Coulomb penetration factor C^2 is equal to 0.5 [25]. Thus, careful analysis is required at small excess energies, where the Coulomb interaction dominates.

the η'-proton scattering length $|Re(a_{\eta' p})| < 0.8$ fm, as estimated in reference [2]. In deriving this η'-proton estimate the dashed line in Figure 2 was assumed for the proton-proton FSI.

Different prescriptions for the proton-proton enhancement factors will obviously give different results for $|M_0|$. Figure 4 for example shows $|M_0|$ for the various meson production channels, as extracted when using the distribution of $|M_{pp\to pp}|^2$ depicted in Figure 2 as the dotted line. The $|M_0|$ behaviour for π^0 and η' mesons is now qualitatively different from that shown in Figure 3, and would indicate an unreasonably strong attractive interaction between these mesons and the proton. Results for $|M_0|$ obtained with $|M_{pp\to pp}|^2$ as described by Druzhinin et al. [28] (dashed line in Figure 2), which is close to that presented in Figure 3, can be found in [34].

To minimize ambiguities that result from uncertainties in the proton-proton scattering amplitude, we consider the ratio $|M_0^{\eta(\eta')}|/|M_0^{\pi^0}|$. At first order the integral of $|M_{pp\to pp}|^2$ over phase space is independent of the meson produced. The transition amplitude for η and η' production $|M_0^{\eta(\eta')}|$ is therefore normalized to the one for the π^0 production $|M_0^{\pi^0}|$; this should be independent of the model used for the determination of $|M_{pp\to pp}|^2$, and will allow an estimate of the relative strength of the π^0-proton and $\eta(\eta')$-proton interactions. Indeed, we found that within errors the ratio $|M_0^{\eta(\eta')}|/|M_0^{\pi^0}|$ does not depend on the model used for $|M_{pp\to pp}|^2$. As an example, in Figure 5 we show this ratio as obtained from the amplitude $|M_{pp\to pp}|^2$ presented as the dotted line in Figure 2. Figure 5a shows an increasing strength of $|M_0|$ for the η production at low V_{ps}, indicating a strong η-proton FSI, as was discussed previously for the cross section ratio by Calén et al. [6]. Note also that the ratio for the η' meson is constant over the phase space range considered (Figure 5b). This observation, and the fact that theoretical calculations predict that the primary production amplitude is constant to within a few per cent [16,35] independent of the mechanism assumed, allows us to conclude that the η'-proton scattering parameters are comparable to the π^0-proton ones. The η'-proton scattering length is therefore about 0.1 fm, similar to the π^0-proton scattering length.

Acknowledgements

One of the authors (P.M.) acknowledges financial support from the Forschungszentrum Jülich and the Foundation for Polish Science. We would like to thank T. Barnes for his careful reading of the manuscript. This research project was supported in part by the BMBF (06MS881I), the Bilateral Cooperation between Germany and Poland represented by the Internationales Büro DLR for the BMBF (PL-N-108-95) and by the Komitet Badań Naukowych KBN, and by FFE grants (41266606 and 41266654) from the Forschungszentrum Jülich.

REFERENCES

[1] J. Smyrski et al., Phys. Lett. **B 474** (2000) 182.

[2] P. Moskal et al., Phys. Lett. **B 474** (2000) 416.

[3] F. Hibou et al., Phys. Lett. **B 438** (1998), 41.

[4] P. Moskal et al., Phys. Rev. Lett. **80** (1998) 3202.

[5] H. Calén et al., Phys. Rev. Lett. **79** (1997), 2642.

[6] H. Calén et al., Phys. Lett. **B 366** (1996), 39.

[7] E. Chiavassa et al., Phys. Lett. **B 322** (1994), 270.

[8] A. M. Bergdolt et al., Phys. Rev. **D 48** (1993), R2969.

[9] H. O. Meyer et al., Nucl. Phys. **A 539** (1992) 633.

[10] A. Bondar et al., Phys. Lett. **B 356** (1995) 8.

[11] A. Sibirtsev and W. Cassing, Few-Body Systems Suppl. **99** (1999) 1.

[12] H. Machner, J. Haidenbauer, J. Phys. **G 25** (1999) R231.

[13] V. Bernard, N. Kaiser, Ulf-G. Meißner, Eur. Phys. J. **A 4** (1999), 259.

[14] E. Byckling, K. Kajantie, *Particle Kinematics*, John Wiley & Sons Ltd. (1973)

[15] C. Hanhart, K. Nakayama, Phys. Lett. **B 454** (1999) 176.

[16] K. Nakayama et al., Phys. Rev. **C 61** (2000) 024001.

[17] G. Fäldt and C. Wilkin, Phys. Lett. **B 382** (1996), 209.

[18] K.M. Watson, Phys. Rev. **88** (1952), 1163

[19] A. Moalem et al., Nucl. Phys. **A 589** (1995) 649.

[20] B. J. Morton et al., Phys. Rev. **169** (1968) 825.

[21] H. A. Bethe, Phys. Rev. **76** (1949) 38.

[22] J. P. Naisse, Nucl. Phys. **A 278** (1977), 506.

[23] H. P. Noyes, H. M. Lipinski, Phys. Rev. **C 4** (1971), 995.

[24] H. P. Noyes, Ann. Rev. Nucl. Sci. **22** (1972), 465.

[25] J.D. Jackson, J.M. Blatt, Rev. of Mod. Phys. **22** (1950), 77

[26] R.A. Arndt et al., Phys. Rev. **C 56** (1997) 3005.
The Virginia Tech Partial-Wave Analysis Facility (SAID)
http://said.phys.vt.edu/said_branch.html

[27] V.G.J. Stoks et al., Phys. Rev. **C 48** (1993) 792.

[28] B. L. Druzhinin, A. E. Kudryavtsev, V. E. Tarasov, Z. Phys. **A 359** (1997), 205.

[29] R. Shyam, U. Mosel, Phys. Lett. **B 426** (1998) 1.

[30] M.L. Goldberger, K.M. Watson, Collision Theory, Wiley, New York, 1964.

[31] A. Moalem et al., Nucl. Phys. **A 600** (1996), 445.

[32] D. Sigg et al., Nucl. Phys. **A 609** (1996) 269, and Nucl. Phys. **A 617** (1997) 526

[33] A. M. Green, S. Wycech, Phys. Rev. **C 55** (1997), R2167.

[34] P. Moskal et al., 4th International Conference on Physics at Storage Rings, Bloomington, Indiana, USA, 1999, in press by American Institute of Physics.
http://ikpe1101.ikp.kfa-juelich.de/

[35] E. Gedalin, A. Moalem, L. Razdolskaja, Nucl. Phys. **A 650** (1999), 471.

[36] H. O. Meyer et al., Phys. Rev. Lett. **65** (1990), 2846.

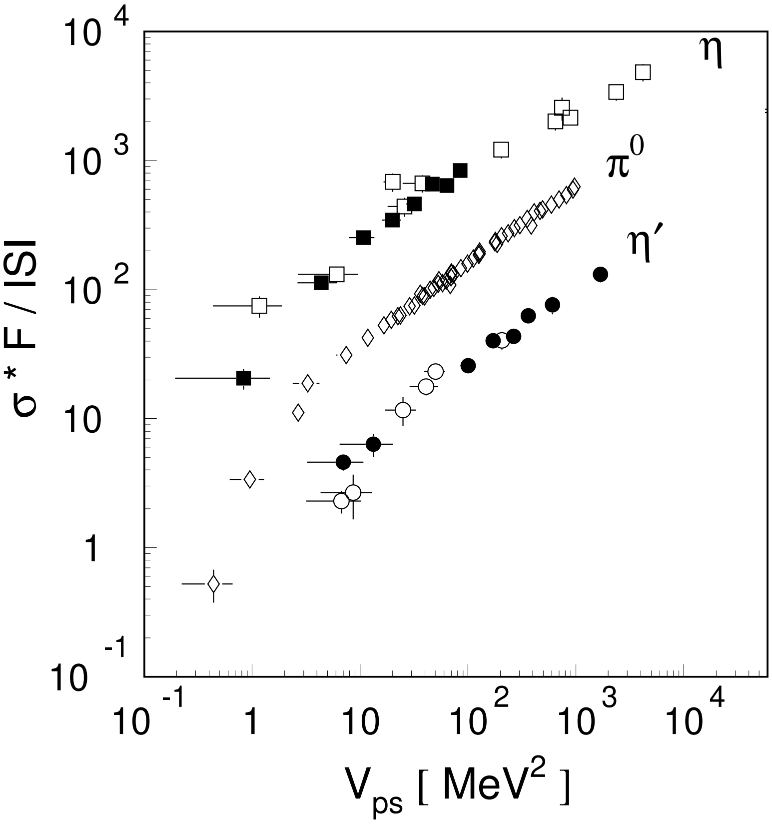

FIG. 1. Total cross section multiplied by the flux factor F and divided by the initial-state-iteraction reduction factor ISI versus the available phase space volume for the reactions $pp \to pp\eta$ (squares [1,3,6–8]), $pp \to pp\pi^0$ (diamonds [9,10,36]), and $pp \to pp\eta'$ (circles [2–4]). The filled symbols are recent COSY - 11 results [1,2]. Note that an increase in the excess energy from Q = 0.5 MeV to Q = 30 MeV corresponds a growth of phase space volume by about three orders of magnitude.

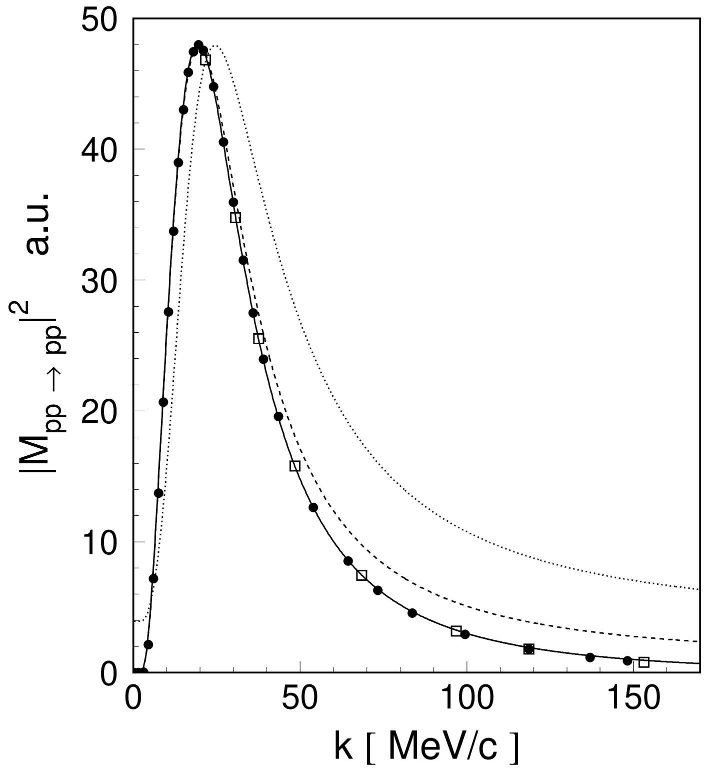

FIG. 2. Square of the proton-proton scattering amplitude versus k, the proton momentum in the proton-proton subsystem. These are references [20,22] (solid line), [28] (dashed line) and [29,30] (dotted line). The filled circles are extracted from [26], and the opened squares are from [27]. The curves and symbols are arbitrarily normalized to be equal at maximum to the result from reference [28], shown as the dashed line.

The presented range of momentum k covers the allowed proton momenta in the excess energy range $Q < 30$ MeV.

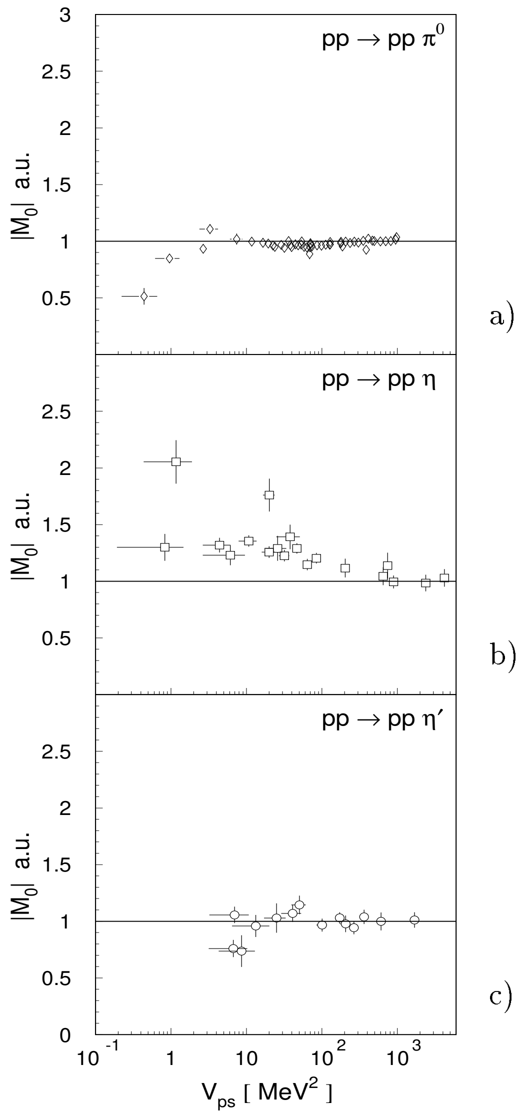

FIG. 3. The quantity $|M_0|$, extracted from the data of references [1–4,6–10,36]. The proton-proton scattering amplitude was calculated according to equation (4), and the phase shifts were computed using equation (5) (see the solid line in Figure 2).

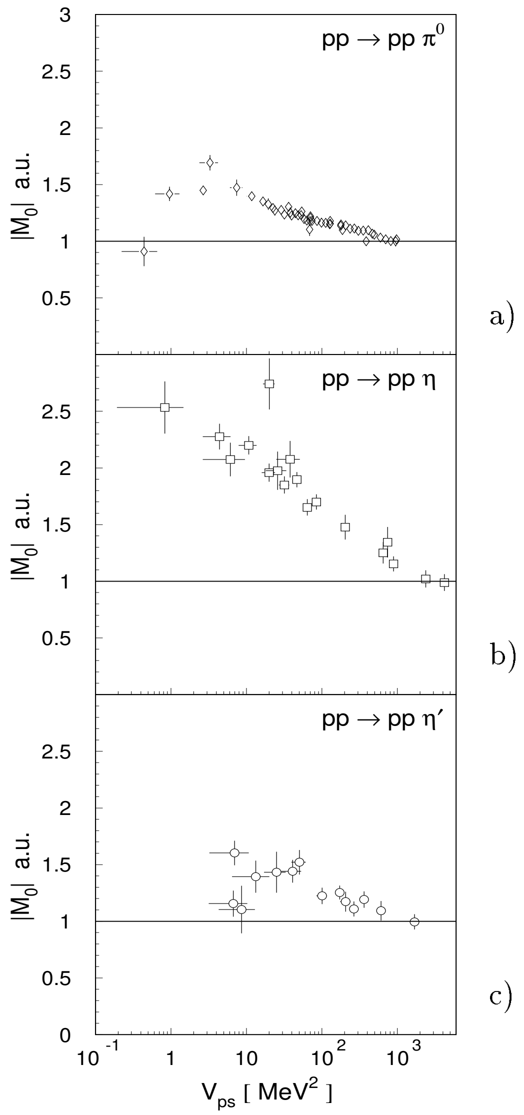

FIG. 4. The quantity $|M_0|$, extracted from the data of references [1–4,6–10,36]. The enhancement due to the pp-FSI was approximated by the inverse of the squared Jost function of references [29,30] (dotted line in Figure 2).

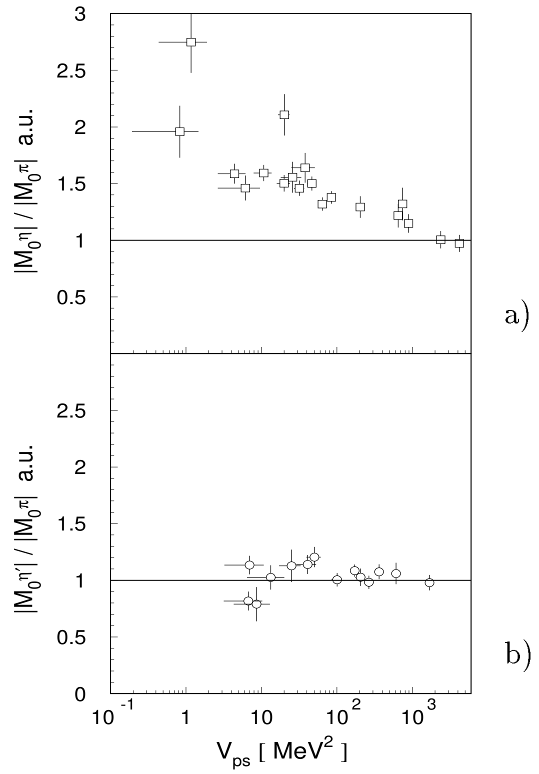

FIG. 5. The ratios of a) $|M_0^\eta|/|M_0^{\pi^0}|$ and b) $|M_0^{\eta'}|/|M_0^{\pi^0}|$ extracted from the data, assuming the pp-FSI enhancement factor depicted by the dotted line in Figure 2. $|M_0^{\pi^0}|$ was calculated by interpolating the points of Figure 4a.