# Chodrow plot and the interaction of $K^+K^-$

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**Abstract.** Measurements of the $pp \to ppK^+K^-$ reaction, performed with the experiment COSY-11 at the Cooler Synchrotron COSY, reveal a significant discrepancy between obtained excitation function and theoretical expectations neglecting interactions of kaons. Thus, the observed enhancement of the data above the predictions may be plausibly assigned to the influence of $K^+K^-$ or $Kp$ interaction. This may manifest itself even stronger in the distributions of the differential cross-sections. Therefore, in order to deepen our knowledge about the low energy dynamics of the ppKK system we investigate population of events for the $pp \to ppK^+K^-$ reaction as a function of the invariant masses of two particle subsystems. In particular generalizations of the Dalitz plot for the four particles proposed by Chodrow and Goldhaber will be presented.

**Keywords:** Kaon, Chodrow, Goldhaber, four particle final state, Kaon-Antikaon interaction
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## INTRODUCTION

The strength of the kaon-antikaon and kaon-nucleon interaction is a crucial quantity for many physics topics. It is for example, an important parameter in the ongoing discussion on the nature of the scalar resonaces $a_0(980)$ and $f_0(980)$, in particular for their interpretation as a $K\bar{K}$ molecules [1, 2, 3]. It is also important in view of discussions on the structure of the excited hyperon $\Lambda(1405)$, which is considered as a candidate for a possible $KN$ bound state [4]. Furthermore, good understanding of kaon and antikaon interaction with nucleon is essential in many calculations connected with neutron stars [5, 6].
Because kaon targets are still unavailable, the only realistic way to study $K\bar{K}$ interaction is the near threshold kaon pair production in multi particle exit channels like $pp \to ppK^+K^-$. Measurements of the total cross section of the aforementioned reaction were performed near the kinematical threshold [7, 8, 9] by the COSY-11 collaboration [10] at the cooler synchrotron COSY, and for higher energy at $Q$ = 114 MeV the experiment was conducted by the DISTO collaboration [11] at the SATURN accelerator. The results are shown in Fig. 1, which contains also theoretical expectations normalized in amplitude to the experimental point at the excess energy of $Q$ = 114 MeV. One observes that near threshold data points lie significantly above any expectations. Predictions based on the assumption of homogeneous phase space occupation differs from experimental data by two orders of magnitude at $Q$ = 10 MeV (dashed line in Fig. 1). It is also evident that the inclusion of the $pp$-FSI (dashed-dotted line) or calculations within a one-boson exchange model [12] (solid line) cannot fully account for the discrepancy.
The enhancement may be due to the influence of $K^+K^-$ or $Kp$ interaction which was neglected in the calculations. This interaction should manifest itself also in the distributions of the differential cross-sections [13]. Indeed, the comparison of the invariant mass

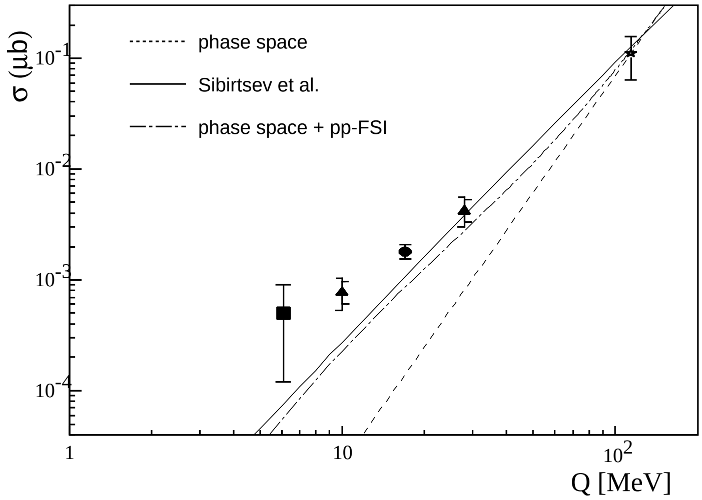


**FIGURE 1.** Total cross section as a function of the exess energy $Q$ for the reaction $pp \rightarrow ppK^+K^-$. The data are form references [7, 8, 11, 14] and the meaning of lines is described in the text.

spectra for the $K^+p$ and $K^-p$ indicates that the $K^-p$ interaction is much stronger that the one between $K^+$ and the proton [8]. As a next step we would like to continue the investigations by extending the analysis into two dimensional distributions.
A possible way to study the interaction is to compare distributions constructed from experimental data to the results of Monte Carlo simulations generated with various parameters of the $K^+K^-$ and $Kp$ interaction. In our investigation we use experimental data obtained from two COSY-11 measurements at excess energies of $Q$ = 10 MeV (27 events) and 28 MeV (30 events) [8]. At present we are, however, at the early stage of the analysis and in this report we will present still preliminary spectra which we intend to use for the evaluation of discussed interactions in the near future. A significant effect observed for the excitation function for the $pp \rightarrow ppK^+K^-$ reaction encourages us to take the effort to carry out the analysis in spite of the fact that the available statistics is quite low.

## DALITZ PLOT - CONVENIENT METHOD OF ANALYSIS FOR THREE-PARTICLE FINAL STATES

For three particles in the final state the process of analysing elementary-particle-reaction data by plotting them in the space of two internal parameters is well known. It was originated by Dalitz in a nonrelativistic application. In the original paper [15] Dalitz let the distances to the sides of an equilateral triangle be the energies of the three particles in the centre-of-mass frame. The sum of distances from a point within the triangle to its sides is constant and equal to the height, which represents the total energy. Therefore, the interior points fulfil four-momentum conservation and represent energy partitions. The relativistic extension of Dalitz's analysis was given by Fabri [16]. More instructive coordinates than energies are the squared invariant masses of the two-body subsystems [17]. Using such coordinates we obtain event distribution bounded by well defined smooth closed curve [18]. The area of the Dalitz Plot is proportional to the phase space volume. Moreover, in case of no dynamics and the absence of any final state interaction, the occupation of the Dalitz plot would be fully homogeneous because the creation in any phase space interval would be equally probable. Thus, final state interaction shows up as a modification of the event density on Dalitz plot. Such effect was observed experimentally e.g. by the COSY-11 collaboration for the $pp \rightarrow pp\eta$ reaction [19].

## GENERALIZATION OF THE DALITZ PLOT FOR FOUR-PARTICLE FINAL STATES

In case of a four body in final state the analysis is more complex, because one need five variables to fully describe a relative movement of particles. Nevertheless, there are many different types of generalization of the Dalitz plot for four-particle final states, which make possible to study interaction between particles in the exit channel. In this contribution we present two convenient generalizations proposed by Chodrow [20] and Goldhaber [21, 22], which we use for studing of the $K^+K^-$ or $Kp$ interaction. However, there exist many other approaches as described e.g. by Nyborg [18].

*Chodrow Plot.* Consider four particles with masses $m_i$ and total energy $E$ in centre-of-mass frame. The probability of a reaction yielding a state with $i$th particle in momentum range $d^3p_i$ is:

$$d^{12}P = d^3p_1 d^3p_2 d^3p_3 d^3p_4 \frac{1}{16E_1E_2E_3E_4} \delta^3\left(\sum_{j=1}^{4} \vec{p}_j\right) \delta\left(\sum_{j=1}^{4} E_j - E\right) |M|^2 \quad (1)$$

where $E_i = \sqrt{\vec{p}_i^{\,2} + m_i^2}$ is an energy of the $i$th particle ($c$=1) and $M$ denotes the invariant matrix element for the process. In his work [20] Chodrow assumed, that $M$ depends only on energies of the particles. The probability expressed in eq.(1) is then given by:

$$d^3P = dE_1 dE_2 dE_3 \, |M|^2 \, min\left(|\vec{p}_1|, |\vec{p}_2|\right). \quad (2)$$

It is then possible to analyse the resonanses by plotting event distribution in $E_1E_2$-plane. However the analysis won't be easy due to the factor $min\left(|\vec{p}_1|, |\vec{p}_2|\right)$ [20]. This difficulty can be avoided if in final state particle 1 and 2 are identical. This condition is fulfilled in the case of the $ppK^+K^-$ system. In this case analysis can be confined to region of $E_1E_2$-plane defined by condition $E_1 < E_2$. Then from eq.(2) one gets:

$$d^3P = 32\pi^2 \, |M|^2 \, dF_1 dE_2 dE_3, \quad (3)$$

where $dF_1 = \sqrt{E_1^2 - m_1^2} dE_1$. This implies that $F_1 = \frac{1}{2}\left[E_1\sqrt{E_1^2 - m_1^2} - m_1^2 cosh^{-1}\left(\frac{E_1}{m_1}\right)\right]$. The distribution of events can then be plotted in the $F_1E_2$-plane, and resonances may be directly read off the plot. Like in case of three particle final states the physically allowed region on Chodrow plot is bounded by well defined curve, but the event density is not

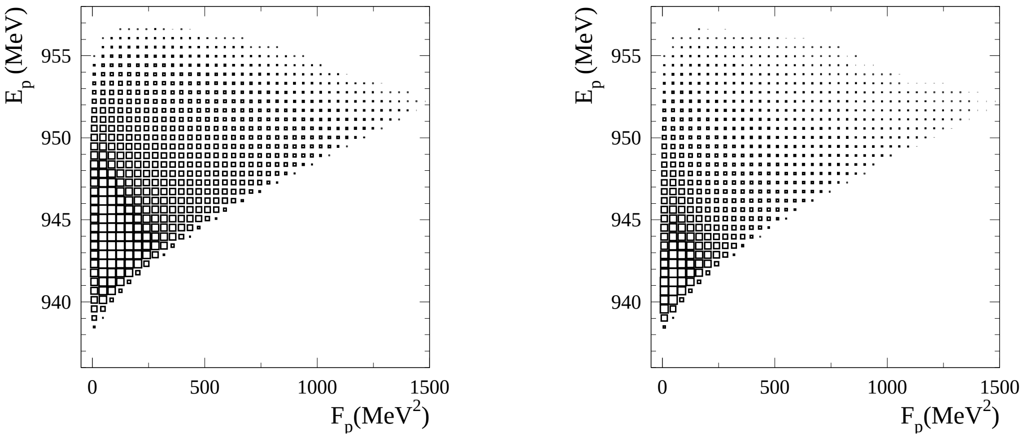


**FIGURE 2.** Monte Carlo simulations of the $pp \rightarrow ppK^+K^-$ reaction at $Q$ = 28 MeV: (left) Chodrow plot for the homogeneously populated phase space; (right) Phase space density distribution modified by proton-proton final state interaction.

homogeneous and the area of the plot is not proportional to the phase space volume. Examples of the Chodrow plots are shown in Fig. 2.

*Goldhaber Plot.* According to Nyborg several others extensions of Dalitz plot can be obtained if one assumes, that the matrix element $M$ depends only on invariant masses of two- and three particle subsystems [18]. Eq.(1) can be now integrated over spatial orientations of the four particle system, and expressed as a distribution in some choice of five independent invariant masses. A very convenient set of variables is: $M_{12}^2$, $M_{34}^2$, $M_{14}^2$, $M_{124}^2$, $M_{134}^2$. The result is:

$$d^5P = \frac{\pi^2}{8E^2}\,|M|^2\,\frac{1}{\sqrt{-B}}\;dM_{12}^2\;dM_{34}^2\;dM_{14}^2\;dM_{124}^2\;dM_{134}^2 \tag{4}$$

where $B$ is a function of above-mentioned invariant masses, which exact form can be found in Nyborg's work [18]. Let suppose that $M$ depends at most on $M_{12}^2$, $M_{34}^2$, and $M_{124}^2$, which correspond to situation were only two two-particle or one three-particle resonances are present [18]. Eq.(4) can be now integrated over $M_{14}^2$, $M_{134}^2$, and the result gives the distribution of events:

$$d^3P = \frac{\pi^3}{8E^2M_{12}^2}\,|M|^2\,g\left(M_{12}^2, m_1^2, m_2^2\right)\;dM_{12}^2\;dM_{34}^2\;dM_{124}^2, \tag{5}$$

where $g\left(M_{12}^2, m_1^2, m_2^2\right) = \sqrt{\left[M_{12}^2-(m_1+m_2)^2\right]\left[M_{12}^2-(m_1-m_2)^2\right]}$. The projection of the physical region on the $(M_{12},\ M_{34})$-plane gives a right isosceles triangle within which the area is not proportional to the phase space volume (Fig. 3). Analysis of the event distribution on the $(M_{12},\ M_{34})$-plane was performed for the first time by Goldhaber in 1962 [21, 22]. It is worth mentioning that the event density on Goldhaber plot is

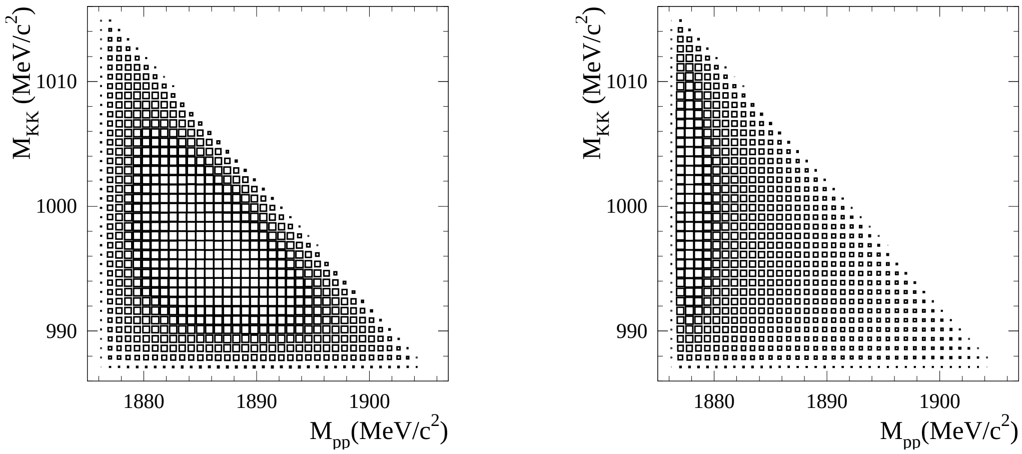


**FIGURE 3.** Monte Carlo simulations of the $pp \to ppK^+K^-$ reaction at $Q$ = 28 MeV: (left) Goldhaber plot for a homogeneously populated phase space; (right) Phase space density distribution modified by proton-proton final state interaction.

not homogeneous and goes to zero on entire boundary of the plot given by following equations: $M_{12} + M_{34} = E$, $M_{12} = m_1 + m_2$, $M_{34} = m_3 + m_4$ [18].

## PRESENT STATUS OF ANALYSIS

So far we conducted Monte Carlo simulations of the $pp \to ppK^+K^-$ reaction using a FORTRAN-based code, called GENBOD [23]. It generates four-momentum vectors of the outgoing particles in the centre of mass frame with the homegeneous distribution in the phase space. The total centre of mass energy as well as the number and masses of the particles are specified by the user. The simulations were first made assuming that there is no final state interaction, then the *pp*-FSI was included [24]. The *pp*-FSI was taken into account as a weights proportional to the inverse of a squared Jost-function of the Bonn potential [25]. The simulated distributions for the Chodrow and Goldhaber plots are shown in Fig. 2 and 3. In order to compare these spectra with the experimental distributions we need to correct them for the acceptance and detection efficiency of the COSY-11 facility. This will be a next step of the investigations which we intend to perform in the near future. An example of the experimental event distribution on the Goldhaber plot is shown in Fig. 4. Even though the statistics is low one recognises cleary that the density distribution is different from results of simulations shown in Fig. 3. However, at present, before the acceptance corrections, any interpretation would be prematured.

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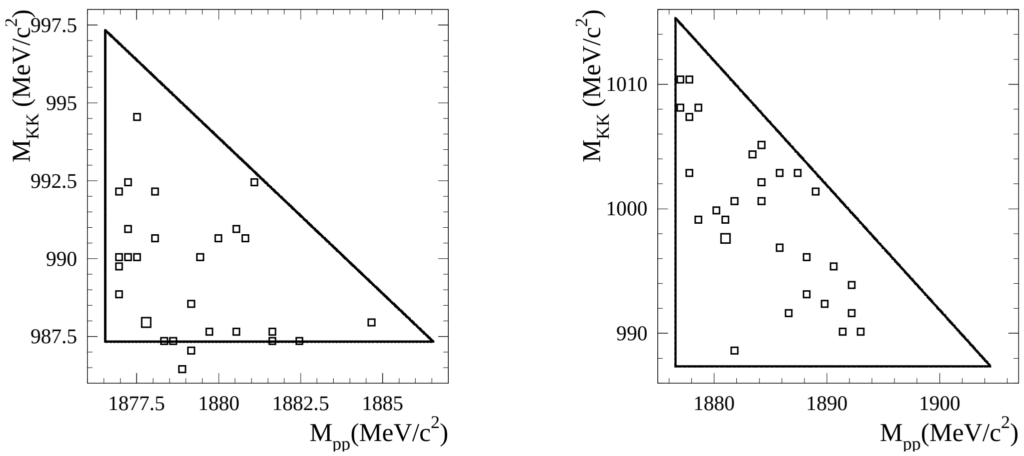


**FIGURE 4.** Goldhaber Plots obtained from measurements [8] at $Q$ = 10 MeV (left) and $Q$ = 28 MeV (right). Solid lines show boundaries of the physically allowed region at appropriate excess energies.

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